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A. N. Cherepanov and V. I. Yakovlev

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A SELF-SIMILAR SOLUTION TO THE PROBLEM OF THE EXPANSION OF A CYLINDRICAL COLUMN OF CONDUCTING GAS IN A LONGITUDINAL MAGNETIC FIELD

A. N. Cherepanov and V. I. Yakovlev

ABSTRACT. Nonstationary radial motion of an infinitely long cylindrical column of conducting gas is considered in a time-variable longitudinal magnetic field. /14*

On the assumption of the proportionality of static pressure of the plasma on the boundary of the column to the external magnetic pressure, accurate solutions to a system of equations of magnetohydrodynamics are found by the method of variable division. Some numerical calculations are performed and the energy characteristics are computed for the interaction process. Dependences of the ratio of the useful work performed by the gas in an infinite time interval to the initial energy of the column on the magnetic Reynolds number are presented. It should be noted that a similar method was employed in [1], where, in addition to averaging the cross-sectional temperature, the inertia of the medium is disregarded; the calculation of inertia leads to the requirement of proportionality of the static pressure to the magnetic pressure on the boundary of the column.

A physically similar model can, for example, be interpreted as the expansion of a compressed conducting gas column in a nonconducting uncompressed fluid situated in a permeable cylinder with a certain radius R which is infinite with respect to its axis of symmetry. Then the requirement of proportionality of static pressure to magnetic pressure is reduced to the condition of external pressure variation on the boundary of a permeable cylinder with radius R according to a specific law, which may be easily determined.

We shall make the following assumptions.

(1) The conductivity of the gas is finite and is determined by the temperature

$$\frac{\sigma}{\sigma_0} = \left(\frac{T}{T_0}\right)^n \quad (n \geqslant 0) \tag{0.1}$$

(2) The gas is ideal, and viscosity and caloricity are not taken into account.

(3) Displacement currents are disregarded everywhere. In particular, it is considered feasible to assign an arbitrary law of variation in intensity of the magnetic field on the external boundary of an expanding cylindrical column,

*Numbers in the margin indicate pagination in the foreign text.

not considering the electromagnetic waves in the external nonconducting space. The latter is valid if the rate of expansion is slightly less than the velocity of light.

(4) A static pressure proportional to the external magnetic pressure is maintained on the external boundary of the column.

The last requirement is associated with the condition of self-similarity of the problem in the sense of variable division.

1. Fundamental equations. Under assumptions (1)-(3), a system of magnetohydrodynamics equations in a cylindrical coordinate system has the following form:

$$\boxed{\begin{gathered}\frac{\partial H}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(rvH)=\frac{c^2}{4\pi}\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{\sigma}\frac{\partial H}{\partial r}\right)\\ \frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}=-\frac{1}{\rho}\frac{\partial}{\partial r}\left(p+\frac{H^2}{8\pi}\right),\qquad \frac{\partial\rho}{\partial t}=-\frac{1}{r}\frac{\partial}{\partial r}(rv\rho)\\ \rho c_v\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial r}\right)=-\rho p\frac{d}{dt}\left(\frac{1}{\rho}\right)+\frac{c^2}{16\pi^2\sigma}\left(\frac{\partial H}{\partial r}\right)^2\qquad (p=R\rho T)\end{gathered}} \tag{1.1}$$

Here H(r, t) and v(r, t) are the longitudinal and radial vector components H^* and v^*, respectively. There are no other vector components H and v ($d/dz \equiv 0$, $d/d\varphi \equiv 0$). We shall seek a solution which satisfies the condition of proportional expansion, i.e., /15

$$\boxed{v(r,t)=\frac{r}{a(t)}\frac{da}{dt}} \tag{1.2}$$

where $a(t)$ is the unknown law of motion of the boundary of the cylindrical column.

We shall introduce the following notations:

$$\boxed{\begin{gathered}h_1=\frac{H}{H_0},\quad p_1=\frac{p}{H_0^2/8\pi},\quad \theta_1=\frac{T}{T_0},\quad \rho_1=\frac{\rho}{H_0^2/8\pi RT_0},\quad \lambda=\frac{a}{a_0}\\ \xi=\frac{r}{a},\quad \tau=\frac{v_0 t}{a_0},\quad t_0=\frac{a_0}{v_0}=\frac{a_0}{\sqrt{RT_0}},\quad \nu=\frac{c^2 t_0}{4\pi\sigma_0 a_0^2},\quad v_0=\sqrt{RT_0}=\frac{\sqrt{\gamma RT_0}}{\sqrt{\gamma}}\end{gathered}} \tag{1.3}$$

The following scales have been adopted here for dimensionless quantities: H_0 is the field intensity on the boundary of the column at the initial moment of time; a_0 is the initial radius of the column; T_0 is the temperature on the boundary of the column at the initial moment of time; v_0 is the characteristic

*Translator's note: These are printed in boldface in the Russian text.

velocity; σ_0 is the conductivity at temperature T_0. We shall present equations (1.1), (1.2) and (1.3) in the following form:

$$\begin{gathered}
\frac{\partial}{\partial \tau}(\lambda^2 h_1) = \frac{\nu}{\lambda^2}\frac{1}{\xi^2}\frac{\partial}{\partial \xi}\left[\frac{\xi}{\theta_1^n}\frac{\partial}{\partial \xi}(\lambda^2 h_1)\right] \\
\rho_1 \lambda'' \lambda = -\frac{1}{\xi}\frac{\partial}{\partial \xi}(p_1 + h_1^2), \qquad \frac{\partial \rho_1}{\partial \tau} = -\frac{2\rho_1 \lambda'(\tau)}{\lambda(\tau)} \\
\frac{\partial \theta_1}{\partial \tau} = -\varkappa p_1 \frac{\partial}{\partial \tau}\frac{1}{\rho_1} + 2\nu\varkappa \frac{1}{\rho_1 \theta_1^n}\frac{1}{\lambda^2}\left(\frac{\partial h_1}{\partial \xi}\right)^2, \qquad p_1 = \rho_1 \theta_1 \\
\left(\varkappa = \frac{R}{c_v} = \frac{c_p}{c_v} - 1\right)
\end{gathered} \tag{1.4}$$

The third equation of system (1.4) can be integrated. We then obtain

$$\rho_1 = \Phi(\xi) / \lambda^2(\tau) \tag{1.5}$$

Here $\Phi(\xi)$ is a certain function of ξ. Using (1.5) and introducing new unknown functions

$$h(\xi, \tau) = \lambda^2(\tau) h_1(\xi, \tau), \qquad \theta(\xi, \tau) = \lambda^{2\varkappa}\theta_1(\xi, \tau) \tag{1.6}$$

the remaining equations can be written in the following form:

$$\begin{gathered}
\frac{\lambda''}{\lambda(\tau)} = -\frac{1}{\xi \Phi(\xi)}\frac{\partial}{\partial \xi}\left(p_1 + \frac{h^2}{\lambda^4}\right), \qquad \frac{\partial \theta^n}{\partial \tau} = \frac{2n\nu\varkappa}{\lambda^{6-2\varkappa n}}\frac{1}{p_1}\left(\frac{\partial h}{\partial \xi}\right)^2 \\
\frac{\partial h}{\partial \tau} = \frac{\nu}{\lambda^{2-2\varkappa n}}\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\frac{\xi}{\theta^n}\frac{\partial h}{\partial \xi}\right), \qquad p_1 = \frac{\Phi(\xi)\theta}{\lambda^{2+2\varkappa}}
\end{gathered} \tag{1.7}$$

We shall find the particular solution to system (1.7) in the following form:

$$h(\xi, \tau) = T(\tau) Z(\xi), \quad \theta^n(\xi, \tau) = V(\tau) X(\xi), \quad p_1(\xi, \tau) = G(\tau) Y(\xi) \tag{1.8}$$

(i.e., the self-similar solution [1]); we shall then consider that

$$T(0) = 1, \quad V(0) = 1, \quad G(0) = 1 \tag{1.9}$$

It is easy to note that the division of the variables in equations (1.7) is possible under the following condition:

$$\frac{T^2(\tau)}{\lambda^4(\tau) G(\tau)} = \mathrm{const} = 1 \tag{1.10}$$

The constant in this case is equal to 1 by virtue of (1.9) and the initial/16 condition

$$\boxed{\lambda(0) = 1} \tag{1.11}$$

It follows from (1.10) that the ratio of static pressure to magnetic pressure for each given particle in this case is a constant which does not depend on time. This condition is satisfied if a pressure is maintained on the external boundary of the cylindrical column which is proportional to the magnetic pressure (assumption (4)).

After substituting (1.8) into system (1.7), using condition (1.10), and dividing the variables, the following two systems of equations are obtained:

for the functions $T(\tau)$, $V(\tau)$, $G(\tau)$, and $\lambda(\tau)$

$$\boxed{\begin{array}{ll} \dfrac{\lambda''(\tau)}{\lambda(\tau)\,G(\tau)} = \alpha, & \dfrac{V(\tau)\,\lambda^{2-2\varkappa n}}{T(\tau)}\dfrac{dT}{d\tau} = \beta \\ \dfrac{\lambda^{6+2\varkappa n}\,G(\tau)}{T^{2}(\tau)}\dfrac{dV}{d\tau} = \mu, & \dfrac{\lambda^{2+2\varkappa n}\,G(\tau)}{V^{1/n}(\tau)} = \psi \end{array}} \tag{1.12}$$

for the functions $X(\xi)$, $Y(\xi)$, $Z(\xi)$, and $\Phi(\xi)$

$$\boxed{\begin{array}{ll} -\dfrac{1}{\xi\Phi(\xi)}\dfrac{d}{d\xi}\left[Y(\xi) + Z^{2}(\xi)\right] = \alpha, & \dfrac{\nu}{Z(\xi)}\dfrac{1}{\xi}\dfrac{d}{d\xi}\left[\dfrac{\xi}{X(\xi)}\dfrac{dZ}{d\xi}\right] = \beta \\ \dfrac{2n\nu\varkappa}{Y(\xi)\,X(\xi)}\left(\dfrac{dZ}{d\xi}\right)^{2} = \mu, & \dfrac{\Phi(\xi)}{Y(\xi)}X^{1/n}(\xi) = \psi \end{array}} \tag{1.13}$$

Here α, β, μ, and ψ are certain constants. In view of the adopted scalar quantities and the standardizing conditions (1.9) for functions of τ, the boundary conditions for the space functions will be the following:

$$\boxed{Z(\xi)|_{\xi=1} = 1, \quad X(\xi)|_{\xi=1} = 1, \quad Y(\xi)|_{\xi=1} = q} \tag{1.14}$$

where q designates the ratio of static pressure to magnetic pressure on the boundary of the column under consideration. On the basis of (1.9), the constant ψ from the last equations (1.12) and (1.13) should be equal to unity.

2. Integration of the systems of equations obtained. The unknown functions should satisfy not only the systems of equations (1.12), but also an additional condition (1.10), which is necessary for obtaining the considered particular solution; therefore, the constants α, β, and μ cannot be arbitrary. Indeed, let us first examine the systems of equations for the functions of the

variable τ. In place of function $G(\tau)$, in all equations (1.12) we shall substitute its expression by $\lambda(\tau)$ and $T(\tau)$ from (1.10).

Then, for the unknowns $T(\tau)$, $V(t)$, and $\lambda(\tau)$, we will have a system of four equations

$$\begin{gathered} \lambda''(\tau)\,\lambda^3(\tau) = \alpha T^2(\tau), \qquad \frac{d\ln T}{d\tau} = \frac{\beta}{\lambda^{2-2\varkappa n}}\,\frac{1}{V(\tau)} \\ \frac{dV}{d\tau} = \frac{\mu}{\lambda^{2-2\varkappa n}}, \qquad T(\tau) = V^{1/2n}(\tau)\,\lambda^{1-\varkappa}(\tau) \end{gathered} \tag{2.1}$$

The number of equations is greater than the number of unknowns; therefore, the system can be compatible only in the presence of definite relationships between the constants α, β, and μ. In order to obtain these relationships, we shall examine the last three equations of system (2.1); we then have

$$T(\tau) = [\lambda(\tau)]^{\frac{(1-\varkappa)2n\beta}{2n\beta-\mu}}, \qquad V(\tau) = [\lambda(\tau)]^{\frac{(1-\varkappa)2n\mu}{2n\beta-\mu}} \tag{2.2}$$

$$\frac{(1-\varkappa)\,2n}{2n\beta-\mu}\,\lambda'(\tau) = [\lambda(\tau)]^{\frac{2n\beta(2\varkappa n-1)+\mu(1-2n)}{2n\beta-\mu}} \tag{2.3}$$

We shall differentiate the last equation and write it in the following form: /17

$$\lambda''(\tau) = \frac{2n\beta-\mu}{[(1-\varkappa)\,2n]^2}\,[2n\beta(2\varkappa n-1)+\mu(1-2n)]\,[\lambda(\tau)]^{\frac{2n\beta(4\varkappa n-3)+\mu(3-4n)}{2n\beta-\mu}} \tag{2.4}$$

Substituting $T(\tau)$ from (2.2) into the first equation of system (2.1), we obtain another equation for $\lambda(\tau)$

$$\lambda''(\tau) = \alpha\lambda^{\frac{4n\beta(1-\varkappa)}{2n\beta-\mu}-3} \tag{2.5}$$

For compatability of system (2.1), equations (2.4) and (2.5) should be identical. It is necessary to consider the 2 cases separately.

First case: $\alpha \neq 0$. We shall equate the exponents with respect to λ, and also the constant factors in the right-hand sides of these equations (2.4) and (2.5).

As a result, we obtain

$$\beta = \frac{\mu}{\varkappa+2\varkappa n-1}, \qquad \alpha = -\mu^2\,\frac{2n+1}{4n^2(\varkappa+2\varkappa n-1)^2} \tag{2.6}$$

The expression for β was used in this case to derive the expression for α.

Thus, from the three constants α, β, and μ, only one of them will be independent. From equation (2.3) and conditions (1.11) and (2.6), we have

$$\lambda(\tau) = \left[1 + \frac{(n+1)\mu}{n(\varkappa + 2\varkappa n - 1)}\tau\right]^{\frac{2n+1}{2(n+1)}} \qquad \left(\mu = k\,\frac{2n(\varkappa + 2\varkappa n - 1)}{2n+1}\right) \tag{2.7}$$

It follows from equations (2.2) that

$$T(\tau) = \lambda^{\frac{2n}{2n+1}}, \qquad V(\tau) = \lambda^{\frac{2n(\varkappa + 2\varkappa n - 1)}{2n+1}} \tag{2.8}$$

The independent constant μ in (2.7) is expressed by the dimensionless initial velocity $k = \lambda'(0)$.

Second case: α = 0. The identity of equations (2.4) and (2.5) requires that the constant factor in the right-hand side of equation (2.4) be equal to 0. There are two possibilities for this.

The first possibility, $2n\beta - \mu = 0$, leads to a trivial solution, i.e., $\lambda'(\tau) \equiv 0$. The second one, $2n\beta\,(2\varkappa n - 1) + \mu\,(1 - 2n) = 0$, gives

$$\beta = \frac{(2n-1)\mu}{2n(2\varkappa n - 1)} \tag{2.9}$$

Here

$$\lambda(\tau) = 1 + k\tau, \qquad k = \frac{\mu}{2\varkappa n - 1} \tag{2.10}$$

$$T(\tau) = \lambda^{\frac{2n-1}{2n}}, \qquad V(\tau) = \lambda^{2\varkappa n - 1} \tag{2.11}$$

Thus, the system of equations for functions of τ is completely solved.

As follows from (2.8) and (2.11), the solution obtained is characterized by the fact that the external magnetic field (i.e., the magnetic field on the external surface of the column) does not remain constant, but drops as the radius of the column increases. Indeed,

$$h_1(1,\tau) = \frac{T(\tau)}{\lambda^3(\tau)} = \begin{cases} \lambda^{\vartheta} & \text{when } \alpha = 0 \ (\vartheta = -(2n+1)/2n) \\ \lambda^{\theta} & \text{when } \alpha \neq 0 \ (\theta = -(2n+2)/(2n+1)) \end{cases}$$

3. <u>Solution to system of equations (1.13) for space functions</u>. First case: α = 0. From the first equation of system (1.13), when α = 0, and under the boundary conditions (1.14), we have /8

$$Y(\xi) = q_1 - Z^2(\xi), \qquad q_1 = 1 + q \tag{3.1}$$

From the third equation of system (1.13), using (3.1), we obtain

$$X(\xi) = \frac{2n\nu\varkappa}{\mu}\frac{1}{\varphi^2(\xi)}\left(\frac{dZ}{d\xi}\right)^2 = \frac{2n\nu\varkappa}{\mu}\frac{1}{q_1 - Z^2}\left(\frac{dZ}{d\xi}\right)^2 \tag{3.2}$$

We shall substitute this expression into the second equation of system (1.13); using (2.9), we obtain

$$Z_{\xi\xi}'' + B\frac{Z}{q_1 - Z^2}(Z_\xi')^2 - \frac{1}{\xi}Z_\xi' = 0 \qquad \left(B = 2 + \frac{(2n-1)\varkappa}{2\varkappa n - 1}\right) \tag{3.3}$$

The boundary conditions for this equation are expressed in the following form:

$$Z(1) = 1, \qquad \left.\frac{dZ}{d\xi}\right|_{\xi=1} = \left(\frac{\mu}{2n\nu\varkappa}q\right)^{1/2} = \left(\frac{k(2\varkappa n - 1)}{2n\nu\varkappa}q\right)^{1/2} \tag{3.4}$$

The second condition is obtained from the third equation of system (1.13) and the boundary conditions (1.14) for the functions $X(\xi)$ and $Y(\xi)$.

We shall introduce a new independent variable, $x = \ln \xi$, for the solution to equation (3.3); for the function $u(x) = Z(\xi)$, we obtain

$$u'' - 2u' + B\frac{u}{q_1 - u^2}(u')^2 = 0$$

This equation does not contain a clearly independent variable x. Therefore, we shall introduce a new function, $\varphi(u) = u'$; we then obtain the following for the unknown φ:

$$\varphi' + B\frac{u}{q_1 - u^2}\varphi = 2, \qquad \varphi(1) = \left(\frac{k}{\nu}\frac{2\varkappa n - 1}{2\varkappa n}q\right)^{1/2} \tag{3.5}$$

Since $x = 0$ when $\xi = 1$, the boundary condition is obtained from (3.4)

$$u|_{x=0} = Z(\xi)|_{\xi=1} = 1, \qquad u_x'|_{x=0} = \xi Z_\xi'|_{\xi=1} = \left(\frac{k}{\nu}\frac{2\varkappa n - 1}{2\varkappa n}q\right)^{1/2}$$

The solution to equation (3.5) has the following form:

$$\varphi = (q_1 - u^2)^{\frac{B}{2}}\left[2\int_1^u \frac{du}{(q_1 - u^2)^{B/2}} + q^{-\frac{B}{2}}\left(\frac{k}{\nu}\frac{2\varkappa n - 1}{2\varkappa n}q\right)^{1/2}\right] \tag{3.6}$$

Since $\varphi = du/dx$, we find the following from (3.6) and the condition that $u(0) = 1$:

$$x = \frac{1}{2}\ln\left\{\frac{2q^{B/2}}{K}\int_1^u \frac{du}{(q_1 - u^2)^{B/2}} + 1\right\} \qquad \left(K = \left(\frac{k}{\nu}\,\frac{2\varkappa n - 1}{2\varkappa n}\,q\right)^{1/2}\right) \tag{3.7}$$

Hence, the sought solution to equation (3.3) is obtained, which satisfies conditions (3.4)

$$\xi = \left\{\frac{2q^{B/2}}{K}\int_1^Z \frac{d\omega}{[q_1 - \omega^2]^{B/2}} + 1\right\}^{1/2} \tag{3.8}$$

The ratio k/ν which enters here through the expression (3.7) for K is essentially the magnetic Reynolds number, since

$$\frac{k}{\nu} = \frac{4\pi\sigma_0 a_0 a'(t)\,|_{t=0}}{c^2} = R_m \tag{3.9}$$

Having designated the value of Z by Z_0 when $\xi = 0$, from (3.8) and taking into account (3.7), we have /19

$$\int_{Z_0}^1 \frac{d\omega}{(q_1 - \omega^2)^{B/2}} = \frac{1}{2}\left(\frac{k}{\nu}\,\frac{2\varkappa n - 1}{2\varkappa n}\right)^{1/2} q^{\frac{1-B}{2}} \tag{3.10}$$

This gives the dependence of Z_0 on the magnetic Reynolds number R_m. By using this relationship, the expression for $Z(\xi)$ can be written in the following form:

$$\xi = \left(1 - \frac{\psi(Z)}{\psi(Z_0)}\right)^{1/2} \qquad \left(\psi(Z) = \int_Z^1 f(\omega)\,d\omega,\ f(\omega) = \frac{1}{(q_1 - \omega^2)^{B/2}}\right) \tag{3.11}$$

Thus, for the case of $\alpha = 0$, the system of equations for the space functions also can be integrated to the end (the solution to $Y(\xi)$ is given by formula (3.1), by formula (3.2) for $X(\xi)$, and the solution to $\Phi(\xi)$ is obtained from the fourth equation of system (1.13)).

Second case: $\alpha \neq 0$. The constants α, β, and μ, in system of equations (1.13), according to (2.6) and (2.7), are determined as single values by assigning the dimensionless initial velocity $k = \lambda'(0)$. System (1.13) can not be analytically integrated; therefore, we shall reduce it to a form suitable for numerical solution on a computer.

To do this, we express the functions $\Phi(\xi)$ and $X(\xi)$ from the last two equations of system (1.13) by $Y(\xi)$ and $Z(\xi)$; we then have

$$X(\xi) = \frac{2n\nu\varkappa}{\mu}\frac{1}{Y(\xi)}\left(\frac{dZ}{d\xi}\right)^2, \qquad \Phi(\xi) = \left[\frac{2n\nu\varkappa}{\mu}\right]^{-1/n} Y^{1+1/n}(\xi)\left(\frac{dZ}{d\xi}\right)^{-2/n} \tag{3.12}$$

Substituting these expressions into the first two equations of system (1.13), we obtain a system of two equations for the functions $Y(\xi)$ and $Z(\xi)$

$$\begin{gathered} \frac{d}{d\xi}\left[\xi\frac{Y(\xi)}{Z_\xi'}\right] = \frac{2n\varkappa}{\varkappa + 2\pi\varkappa - 1}\xi Z \\ \frac{d}{d\xi}[Y + Z^2] = \frac{(2n+1)(2n\nu\varkappa)^{-1/n}}{4n^2(\varkappa + 2\varkappa n - 1)^2}\mu^{2+1/n}\xi Y^{1+1/n}(Z_\xi')^{-2/n} \end{gathered} \tag{3.13}$$

We shall substitute the independent variable $x = \xi^2$ and shall reduce equation (3.13) to the following form:

$$\begin{gathered} \frac{d}{dx}\frac{Y}{Z_x'} = AZ, \qquad \frac{d}{dx}[Y + Z^2] = Dx^{-1/n}Y^{1+1/n}(Z_x')^{-2/n} \\ A = \frac{2n\varkappa}{\varkappa + 2n\varkappa - 1}, \qquad D = k^2(2n+1)^{-(n+1)/n}\left(\frac{k}{\nu}\right)^{1/n}\left(\frac{\varkappa + 2\varkappa n - 1}{\varkappa}\right)^{1/n} 2^{-\frac{n+2}{n}} \end{gathered} \tag{3.14}$$

We shall introduce new unknown functions

$$\psi(x) = \frac{Y(x)}{Z_x'}, \qquad \varphi(x) = Z_x' \tag{3.15}$$

We then obtain a system of equations for the unknowns $\psi(x)$, $Z(x)$ and $\varphi(x)$ from equations (3.14):

$$\frac{d\psi}{dx} = AZ, \qquad \frac{dZ}{dx} = \varphi, \qquad \frac{d\varphi}{dx} = D\left(\frac{\psi}{x}\right)^{1/n}\varphi^{(n-1)/n} - Z\frac{\varphi}{\psi}(2 + A) \tag{3.16}$$

On the basis of (1.14) and (3.4) we have the following boundary conditions for the unknown functions:

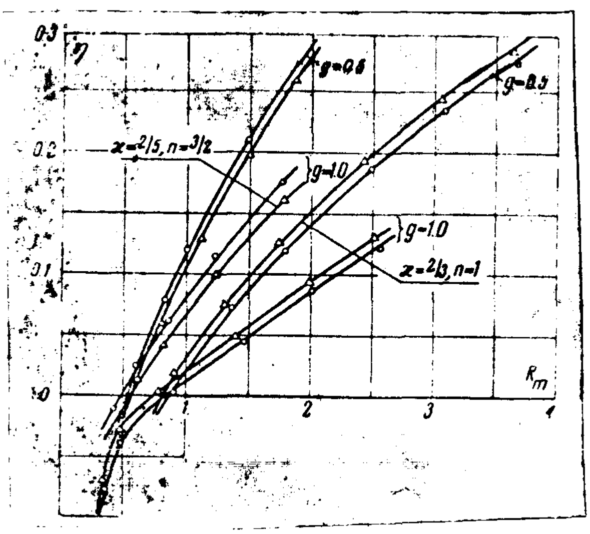
$$Z(x)|_{x=1} = 1, \quad \varphi(x)|_{x=1} = \frac{K}{2}, \quad \psi(x)|_{x=1} = \frac{2q}{K}\left(K - \left(\frac{k}{\nu}\frac{2\varkappa n - 1}{2\varkappa n}q\right)^{1/n}\right) \tag{3.17}$$

Thus, the space functions $Z(\xi)$, $Y(\xi)$, $\Phi(\xi)$, and $X(\xi)$ in the case of $\alpha \neq 0$ /20
are found by integrating system (3.15) using the conditions (3.17).

Some numerical calculations were performed and the energy characteristics of the interaction process were computed, i.e., the amount of work performed during the expansion of the column in opposition to the electrical body forces (EBF), the Joule (heat) losses inside the conducting gas, and the variations in the internal and kinetic energy. As an example, the figure illustrates some values of the coefficient η as a function of the magnetic Reynolds number R_m (the circles correspond to the value of $k = 1.0$; the triangles correspond to $k = 0.5$). The coefficient η is defined as the ratio of useful work performed over time, from $t = 0$ to $t = \infty$, to the initial energy of the column, i.e.,

$$\eta = \frac{A_\infty - Q}{W_0 + U_0}$$

where A_∞ and Q_∞ are the work in opposition to the EBF and the amount of Joule losses in the indicated time interval, while W_0 and U_0 are the kinetic and the internal energy at the initial moment of time, respectively.



The expressions for the energy quantities A_∞, Q_∞, W, and U are not given here since they are easily obtained from the very sense of these quantities.

The dependences obtained for other values of parameters $\varkappa$, n, q, and k in other cases ($\alpha = 0$ and $\alpha \neq 0$) are analogous to those given here, only the permissible interval of change of R_m (at certain $0 \leqslant Z \leqslant 1$) lies in a region of larger values for small q, and therefore, the values of η in this interval are positive everywhere.

It is evident from the graphs that for certain values of R_m the difference $A_\infty - Q_\infty$ becomes negative, although the work A_∞ performed in opposition to the EBF is positive in this case. A similar phenomenon was obtained in [2] for a case when there was no magnetic field inside the column at the initial moment of time, and the intensity of the magnetic field on the boundary of the column was not equal to zero.

The presented data indicate that a similar phenomenon may take place also in the case of continuous initial distribution of the magnetic field inside and on the boundary of the column (the initial distribution of the magnetic field in this case is determined by the function $Z(\xi)$).

It should be emphasized that in this work, as well as in reference [2], all the energy quantities refer to a time interval which begins at a certain "initial" moment, and the process of obtaining this initial state and its energy characteristics is not considered.

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